1)

Martin Kunz

$$h(x) = A\,\theta(x)\,x^{\alpha}\exp(-\beta/x)\exp(-\lambda x)$$

$$A^{-1} = \int_0^{+\infty} x^{\alpha}\exp(-\beta/x)\exp(-\lambda x)\,dx = \left\{ \begin{matrix} y = \lambda x \\ dy = \lambda\, dx \end{matrix} \right\} =$$

$$= \int_0^{+\infty} \frac{1}{\lambda}\, y^{\alpha}\cdot \lambda^{-\alpha}\cdot \exp\left(-\frac{\beta\lambda}{y}\right)\exp(-y)\,dy = \left\{ \beta\lambda = \frac{z^2}{4} \Rightarrow z = \sqrt{|4\beta\lambda|}_{>0} \right\} =$$

$$= \frac{1}{\lambda^{\alpha+1}} \underbrace{\int_0^{+\infty} y^{\alpha}\exp\left(-\frac{z^2}{4y}\right)\exp(-y)\,dy}_{= \frac{z^{\alpha+1}}{2^{\alpha}} K_{\alpha+1}(z)} =$$

$$= \left(\frac{z}{\lambda}\right)^{\alpha+1}\cdot 2^{-\alpha}\cdot K_{\alpha+1}(z) = 2\left(\sqrt{\frac{\beta}{\lambda}}\right)^{\alpha+1} K_{\alpha+1}(2\sqrt{\beta\lambda}) = A^{-1}$$

$$\mathbb{E}X = \{\tilde{h}(x) \sim X\} \overset{!}{=} 1 = A\cdot\int_0^{+\infty} x\cdot h(x)\,dx = A\int_0^{+\infty} x^{\alpha+1}\exp(-\beta/x)\exp(-\lambda x)\,dx =$$

$$= \{\text{stejně jako výše} \dots\} = A\cdot\frac{1}{\lambda^{\alpha+2}}\int_0^{+\infty} y^{\alpha}\dots\,dy = \{-\,{}''\,-\} =$$

$$= A\cdot\frac{1}{\lambda^{\alpha+2}}\,\frac{z^{\alpha+2}}{2^{\alpha+1}}\,K_{\alpha+2}(z) = A\cdot 2\cdot\left(\sqrt{\frac{\beta}{\lambda}}\right)^{\alpha+2} K_{\alpha+2}(2\sqrt{\beta\lambda}) =$$

$$= \sqrt{\frac{\beta}{\lambda}}\cdot\frac{K_{\alpha+2}(2\sqrt{\beta\lambda})}{K_{\alpha+1}(2\sqrt{\beta\lambda})} \overset{!}{=} 1 \iff \underbrace{\frac{K_{\alpha+2}(2\sqrt{\beta\lambda})}{K_{\alpha+1}(2\sqrt{\beta\lambda})} = \sqrt{\frac{\lambda}{\beta}}}_{\text{škálovací rovnice}}$$

$$\frac{K_{1/2}(2\sqrt{\beta\lambda})}{K_{-1/2}(2\sqrt{\beta\lambda})} = \sqrt{\frac{\lambda}{\beta}} \Rightarrow \{K_{-a} = K_a\} \quad 1 = \sqrt{\frac{\lambda}{\beta}}$$

$$\sqrt{\beta} = \sqrt{\lambda} \qquad \beta,\lambda \geq 0$$

$$\underline{\beta = \lambda \text{ pro } \alpha = -3/2}$$

2) $\alpha = \beta = 1/3$ Martin Kunz

$\tau = (1, 1, \dots, 1, 0)$. počet částic $N-1$ … počet buněk ozn. N

$$\mathbb{X}(\tau) = (N-1, 1, 0)$$

$$\langle w | v \rangle \overset{!}{=} 1 \quad (\text{WLOG})$$

$\uparrow$ promítne se v Z_N

$$P(\mathbb{X} = \mathbb{X}(\tau)) = \frac{1}{Z_N} \langle w | \mathbb{D}^{N-1} \mathbb{E} | v \rangle$$

$$\mathbb{D}^{N-1}\mathbb{E} = \mathbb{D}^{N-2}(\mathbb{D} + \mathbb{E}) = \mathbb{D}^{N-1} + \mathbb{D}^{N-2}\mathbb{E} = \dots = \mathbb{D}^{N-1} + \mathbb{D}^{N-2} + \dots + \mathbb{D} + \mathbb{E}$$

$$P(\mathbb{X} = \mathbb{X}(\tau)) = \frac{1}{Z_N}\left(\langle w | \mathbb{D}^{N-1} | v \rangle + \dots + \langle w | \mathbb{D} | v \rangle + \langle w | \mathbb{E} | v \rangle \right) =$$

$$= \frac{1}{Z_N}\left(3^{N-1} + \dots + 3 + 3\right) = \frac{3}{Z_N}\left[\sum_{k=0}^{N-2} 3^k + 1\right] =$$

$$= \left\{ a_m = \sum_{k=0}^{m} q^k \, , \; a_{m+1} - a_m = q^{m+1} \, , \; q \cdot a_m + 1 = a_{m+1} \right.$$

$$\left. \Rightarrow a_m = \frac{q^{m+1} - 1}{q - 1} \right\} =$$

$$= \frac{3}{Z_N} \cdot \frac{3^{N-1} + 1}{3 - 1} = \frac{1}{Z_N} \cdot \frac{3}{2}\left(3^{N-1} + 1\right) = *$$

$$Z_N \approx \frac{(1-2\alpha)^2}{(1-\alpha)^2} \cdot \frac{N}{\alpha^N (1-\alpha)^N} = \frac{\frac{1}{9}}{\frac{4}{9}} \cdot \frac{N}{\left(\frac{1}{3}\right)^N \left(\frac{2}{3}\right)^N} = \frac{1}{4} \cdot \frac{N \cdot (3)^{2N}}{2^N} =$$

$$= \frac{N\,(3)^{2N}}{2^{N+2}}$$

$$* \approx \frac{2^{N-1}\left(3^{N-1} + 1\right)}{3^{2N-2} \cdot N} = \left(\frac{2}{9}\right)^{N-1} \cdot \frac{1}{N}\left(3^{N-1} + 1\right) =$$

$$= \frac{1}{N}\left(\frac{2}{3}\right)^{N-1} + \frac{1}{N}\left(\frac{2}{9}\right)^{N-1} \xrightarrow{N \to +\infty} 0$$

Dumbálková

1) $\varphi(r) = -\ln r$

$$\chi(\vec{r}) = \int_{\mathbb{R}^N} p(\vec{r},\vec{v})\, d\vec{v} = \frac{1}{Z_N} e^{-\beta \sum \varphi(r_k)}\, \Theta(\vec{r})\, \delta(L - \sum r_k)$$

$$\left(p(\vec{r},\vec{v}) = \frac{1}{Z_N} e^{-\frac{\beta}{2}\sum_{k=1}^N (v_k - w)^2} e^{-\beta \sum_{k=1}^N \varphi_k(r_k)} \cdot \Theta(\vec{r}) \cdot \delta(L - \sum_{k=1}^N r_k) \right)$$

$$g(r_1) = \int_{\mathbb{R}^{N-1}} \chi(\vec{r})\, dr_2 \dots dr_N = \frac{1}{Z_N(L)} e^{-\beta\varphi(r_1)}\, \Theta(r_1) \underbrace{\int_{\mathbb{R}^{N-1}} \Theta(r_2)\dots\Theta(r_N)\, e^{-\beta\sum_{k=2}^N \varphi(r_k)}\, \delta(L - r_1 - \sum_{k=2}^N r_k)\, dr_2 \dots dr_N}_{= Z_{N-1}(L-r)}$$

$$\Rightarrow \underline{g(r) = \Theta(r)\, \frac{Z_{N-1}(L-r)}{Z_N(L)}\, e^{-\beta\varphi(r)}}$$

$$\left(Z_N^{(L)} = \int_{\mathbb{R}^N} \Theta(\vec{r})\, e^{-\beta\sum_{k=1}^N \varphi(r_k)}\, \delta(L - \sum_{k=1}^N r_k)\, d\vec{r} \right)$$

• $$\underline{\mathcal{L}[Z_N(L)]} = \int_{\mathbb{R}} Z_N(L)\, e^{-sL}\, dL = \int_{\mathbb{R}}\int_{\mathbb{R}^N} \Theta(\vec{r})\, e^{-\beta\sum\varphi(r_k)}\, \delta(L - \sum r_k)\, d\vec{r}\; e^{-sL}\, dL =$$

$$= \int_{\mathbb{R}^N} \Theta(\vec{r})\, e^{-\beta\sum\varphi(r_k)} \underbrace{\int_{\mathbb{R}} \delta(L - \sum r_k)\, e^{-sL}\, dL}_{\mathcal{L}[\delta(L-\sum r_k)] = e^{-s\cdot\sum_{k=1}^N r_k}}\, d\vec{r} = \int_{\mathbb{R}^N} \prod_{k=1}^N \Theta(r_k)\, e^{-\beta\varphi(r_k)}\, e^{-s\cdot r_k}\, dr_k =$$

$$= \prod_{k=1}^N \int_{\mathbb{R}} \Theta(r_k)\, e^{-\beta\varphi(r_k)}\, e^{-s r_k}\, dr_k = \left(\int_{\mathbb{R}} \Theta(r)\, e^{-\beta\varphi(r)}\, e^{-sr}\, dr \right)^N = \mathcal{L}^N\left[\Theta(r)\, e^{-\beta\varphi(r)}\right] =$$

$$\underset{\varphi = -\ln r}{=} \mathcal{L}^N[\Theta(r)\, r^\beta] = \left(\frac{\Gamma(\beta+1)}{s^{\beta+1}} \right)^N = \underbrace{\frac{\Gamma^N(\beta+1)}{s^{N\beta+N}}}_{F(s)}$$

~~aproximace v sedlovém~~

→ aproximace v sedlovém bodě:

$$\frac{d}{ds}\left[\frac{\Gamma^N(\beta+1)}{s^{N\beta+N}}\, e^{sL} \right] = \Gamma^N(\beta+1)(-N\beta-N)\, \frac{e^{sL}}{s^{N\beta+N+1}} + \frac{\Gamma^N(\beta+1)}{s^{N\beta+N}} \cdot L \cdot e^{sL} \overset{!}{=} 0$$

$$f(x) \doteq \frac{F(\lambda)\, e^{\lambda x}}{\sqrt{2\pi\, |H''(\lambda)|}}$$

$$-(N\beta+N) + L\cdot s = 0 \Leftrightarrow \boxed{s = \frac{N(\beta+1)}{L}} =: \lambda$$

sedlový bod

$$\Rightarrow \underline{Z_N(L) \doteq \frac{1}{D}\, \frac{\Gamma^N(\beta+1)\, e^{\lambda L}}{\lambda^{N\beta+N}}}, \text{ kde } D = \sqrt{2\pi\, |H''|}, \quad H = \ln F(s)$$

$$\Rightarrow \underline{Z_{N-1}(L-r) \doteq \frac{1}{D}\, \frac{\Gamma^{N-1}(\beta+1)\, e^{\mu(L-r)}}{\mu^{(N-1)(\beta+1)}}}, \text{ kde } \mu := \frac{(N-1)(\beta+1)}{L}$$

• pro N velké (tj. $N \doteq N-1$) a $N = L$: $\lambda = \beta+1 = \mu$

$$\Rightarrow g(r) = \Theta(r)\, \frac{\Gamma^{N-1}(\beta+1)\, e^{(\beta+1)(L-r)}}{(\beta+1)^{(N-1)(\beta+1)}} \cdot \frac{(\beta+1)^{N(\beta+1)}}{\Gamma^N(\beta+1)\, e^{(\beta+1)L}} \cdot r^{\beta+1} =$$

$$\underline{\underline{g(r) = \Theta(r)\, \frac{(\beta+1)^{\beta+1}}{\Gamma(\beta+1)}\, e^{-(\beta+1)r}\, r^{\beta+1}}}$$

* 10

vlastnosti

$g(x)$ ... GIG distribuce, ať má plató v 0

$\not\checkmark$ chtělo se drobně víc!

5) $B(x|\sigma) = \frac{1}{\sigma} w\left(\frac{x}{\sigma}\right)$, $w(x) = \frac{1}{Z_B}\Theta(1-|x|)\, e^{\frac{1}{x^2-1}}$

$$N(x,\tau) = \int_{-\infty}^{x} \sum_{k=1}^{N} B(y-\alpha_k|\sigma)\, dy = \int_{-\infty}^{x} \sum_{k=1}^{N} \frac{1}{Z_B}\frac{1}{\sigma}\Theta\left(1-\left|\frac{y-\alpha_k}{\sigma}\right|\right) \cdot e^{\frac{1}{\frac{(y-\alpha_k)^2}{\sigma^2}-1}}\, dy$$

$\Theta(1-|y|) > 0 \Leftrightarrow |y| < 1$

$$\rho(x,\tau) = \frac{\partial N(x,\tau)}{\partial x} = \sum_{k=1}^{N} \frac{1}{Z_B}\frac{1}{\sigma}\Theta\left(1-\left|\frac{x-\alpha_k}{\sigma}\right|\right) e^{\frac{1}{\left(\frac{y-\alpha_k}{\sigma}\right)^2-1}}$$

$$\underline{\left(\lim_{\sigma\to0^+}\rho(x,\tau), \varphi(x)\right)} \overset{DEF}{=} \lim_{\sigma\to0^+}\left(\rho(x,\tau),\varphi(x)\right) = \frac{1}{Z_B}\lim_{\sigma\to0^+}\int_{\mathbb{R}}\sum_{k=1}^{N}\frac{1}{\sigma}\Theta\left(1-\left|\frac{x-\alpha_k}{\sigma}\right|\right) e^{\frac{1}{\left(\frac{y-\alpha_k}{\sigma}\right)^2-1}} \cdot \varphi(x)\, dx$$

$$= \left| \begin{array}{c} \frac{x-\alpha_k}{\sigma} = y \quad \to x = \sigma y + \alpha_k \\ dx = \sigma\, dy \end{array} \right| = \frac{1}{Z_B}\lim_{\sigma\to0^+}\int_{\mathbb{R}}\sum_{k=1}^{N}\Theta(1-|y|)\, e^{\frac{1}{y^2-1}} \cdot \varphi(\sigma y+\alpha_k)\, dy$$

$$= \frac{1}{Z_B}\sum_{k=1}^{N}\lim_{\sigma\to0^+}\int_{-1}^{1} e^{\frac{1}{y^2-1}}\cdot\varphi(\sigma\cdot y+\alpha_k)\, dy = \left| \left|e^{\frac{1}{y^2-1}}\cdot\varphi(\sigma y+\alpha_k)\right| \overset{\varphi\in\mathcal{D}}{\le} K\cdot e^{\frac{1}{y^2-1}} \in \mathcal{L}(-1,1) \right| =$$

$\to$ můžu zaměnit lim a $\int$

$$= \frac{1}{Z_B}\sum_{k=1}^{N}\int_{-1}^{1} e^{\frac{1}{y^2-1}}\varphi(\alpha_k)\, dy = \sum_{k=1}^{N}\varphi(\alpha_k)\underbrace{\int_{\mathbb{R}}\frac{1}{Z_B}\Theta(1-|y|)\, e^{\frac{1}{y^2-1}}\, dy}_{\int_{\mathbb{R}} w(y) = 1} = \sum_{k=1}^{N}\varphi(\alpha_k) =$$

$$= \underline{\left(\sum_{k=1}^{N}\underset{\delta(x-\alpha_k)}{\underset{\shortparallel}{\delta_{\alpha_k}(x)}}\, \varphi(x)\right)} \Rightarrow \underline{\underline{\lim_{\sigma\to0^+}\rho(x,\tau) = \sum_{k=1}^{N}\delta(x-\alpha_k)}}$$

$\checkmark$

$V_K \dots$ rychlost kinem. vlnění

$$V_k = \frac{dI}{d\rho} = 150\left(1 - \frac{\rho}{30}\right) e^{-\rho/30}$$ & hledáme extrém (minimum)

$$\frac{dV_k}{d\rho} = \frac{d^2 I}{d\rho^2} = 150\left(-\frac{1}{30} - \frac{1}{30} + \frac{\rho}{900}\right) e^{-\rho/30} \stackrel{!}{=} 0$$

$$\frac{\rho}{900} = \frac{2}{30}$$

$$\rho = \frac{2 \cdot 900}{30} = 60 \text{ voz./km}$$

$$V_{max} = \lim_{\rho \to 0_+} V = \lim_{\rho \to 0_+} \frac{I}{\rho} = \lim_{\rho \to 0_+} 150\, e^{-\rho/30} = 150 \text{ km/h}$$