I. $$y'' - \frac{6+4x^2}{x+x^3}y' + 6\,\frac{2+x^2}{x^2+x^4}y = 0$$

II. $$y' - \frac{3}{x}y = 0 \quad \Big/ \cdot \frac{1}{x^3} = \text{I.F.}$$

$$\left(y \cdot \frac{1}{x^3}\right)' = 0$$

$$y = C \cdot x^3 \qquad \Omega_{II} = [x^3]_\lambda \ \& \ \dim(\Omega_{II}) = 1$$

$$\dim(\Omega_I \cap \Omega_{II}) = 1 \Rightarrow x^3 \in \Omega_I$$

Snížení řádu I. rovnice:

$$y(x) = x^3 \cdot v(x)$$
$$y' = 3x^2 v + x^3 v'$$
$$y'' = 6xv + 6x^2 v' + x^3 v''$$

Dosazení:

$$6xv + 6x^2v' + x^3v'' - \frac{6+4x^2}{x+x^3}\left(3x^2v + x^3v'\right) + 6\,\frac{2+x^2}{x^2+x^4}x^3v = 0$$

$$x^3v'' + \left(6x^2 - \frac{6+4x^2}{1+x^2}x^2\right)v' = 0$$

$$x^3v'' + \frac{6x^2+6x^4-6x^2-4x^4}{1+x^2}v' = 0$$

$$v'' + \frac{2x}{1+x^2}v' = 0 \quad \Big/ \cdot (1+x^2) = \text{I.F.}$$

$$\left(v' \cdot (1+x^2)\right)' = c'$$

$$v' = \frac{c}{1+x^2}$$

$$v = C \cdot \operatorname{arctg}(x) + D$$

$$y = Cx^3 \operatorname{arctg}(x) + Dx^3$$

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In[3]:= Sum[(-1)^n*n/(2n+1)!*x^{n-1}, {n, 0, Infinity}]
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$$\text{Out[3]}= \left\{\frac{\sqrt{x}\,\mathrm{Cos}[\sqrt{x}]-\mathrm{Sin}[\sqrt{x}]}{2x^{3/2}}\right\}$$

$$s(x) = \sum_{n=1}^{\infty} (-1)^n \frac{n}{(2n+1)!} x^{n-1}$$

$$S(x) = \int s(x)\,dx = \sum_{n=1}^{\infty} (-1)^n \frac{1}{(2n+1)!} x^n + C =$$

$$= \frac{1}{\sqrt{x}} \sum_{n=1}^{\infty} (-1)^n \frac{1}{(2n+1)!} (\sqrt{x})^{2n+1} + C =$$

$$= \frac{1}{\sqrt{x}} \left(\sin(\sqrt{x}) - \sqrt{x} \right) + C =$$

$$= \frac{1}{\sqrt{x}} \cdot \sin(\sqrt{x}) + C - 1$$

$$s(x) = S'(x) = \frac{\frac{1}{2\sqrt{x}} \cos(\sqrt{x}) \cdot \sqrt{x} - \frac{1}{2\sqrt{x}} \sin(\sqrt{x})}{x} = \underline{\frac{\sqrt{x} \cdot \cos(\sqrt{x}) - \sin(\sqrt{x})}{2x^{3/2}}}$$

pro $x > 0$

Pro obecné $x \in \mathbb{R} \setminus \{0\}$:

$$s(x) = \frac{\sqrt{x} \cdot \cos(\sqrt{x}) - \sin(\sqrt{x})}{2x^{3/2}} \qquad x \neq 0$$

$\Leftarrow \cos(it) = \cosh(t)$
$\sin(it) = \sinh(t)$

$$= -\frac{1}{6} \qquad x = 0$$