A1)

$$\int_{\mathbb{R}} g(x)\,dx = \int_2^{+\infty} A\cdot e^{-ax}dx = A\left[-\frac{1}{a}e^{-ax}\right]_2^{+\infty} = \frac{A}{a}e^{-2a} \overset{!}{=} 1$$

$$A = a\cdot e^{2a}$$

$$E(X) = A\int_2^{+\infty} x\cdot e^{-ax}dx = \left| \begin{matrix} u = x & v' = e^{-ax} \\ u' = 1 & v = -\frac{1}{a}e^{-ax} \end{matrix} \right| =$$

$$= A\left[-\frac{x}{a}e^{-ax}\right]_2^{+\infty} + \frac{A}{a}\int_2^{+\infty} e^{-ax}dx = A\frac{2}{a}e^{-2a} + \frac{1}{a} = 2 + \frac{1}{a}$$

$$g(x) = a\,\Theta(x-2)\cdot e^{-(a-2)x} \quad ; \quad a>0$$

A2)

$$E = \{1, 2, 3, \dots, 49\} = \widehat{49}$$

$$\mathrm{card}(E) = 49$$

$$A = \{7, 14, 21, 28, 35, 42, 49\} \Rightarrow P[X\in A] = \frac{7}{49} = \frac{1}{7}$$

$$F(28\tfrac{1}{4}) = P[X < 28\tfrac{1}{4}] = P[X\in\{1,2,3,\dots,28\}] = \frac{28}{49} = \frac{4}{7}$$

A3)

$$C = \langle 3, 8) \quad \& \quad D = \langle 5, 9)$$

$$C\cup D = \langle 3, 9) \quad \& \quad C\setminus D = \langle 3; 5)$$

$$P[X\in C\cup D] = P[X\in\langle 3, 9)] = F(9) - F(3) =$$

$$= \left(1-\frac{1}{64}\right) - \left(1-\frac{49}{64}\right) = \frac{48}{64} = \frac{3}{4}$$

$$P[X\in C\setminus D] = F(5) - F(3) = \left(1-\frac{25}{64}\right) - \left(1-\frac{49}{64}\right) = \frac{24}{64} = \frac{3}{8}$$

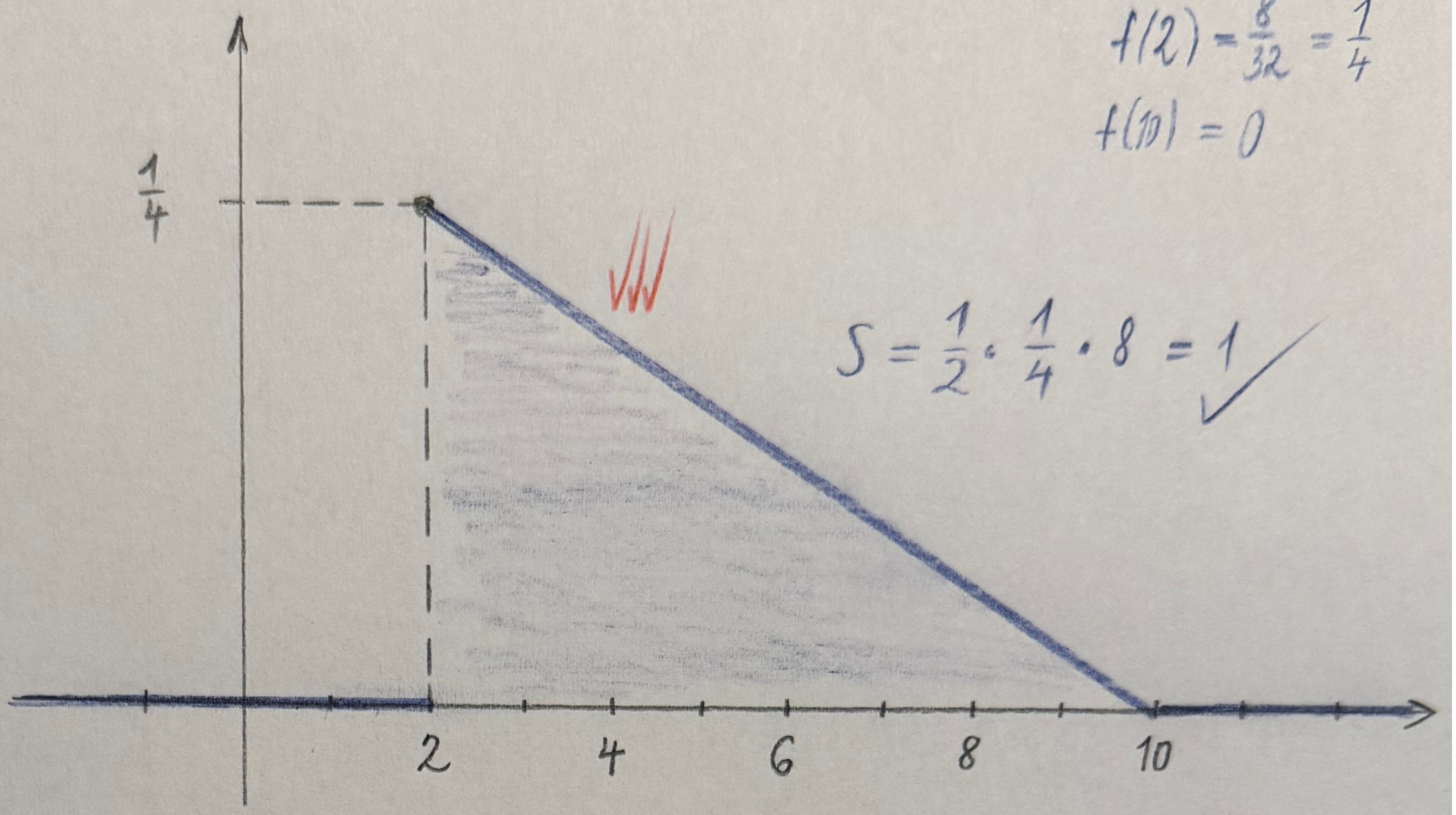
$f(x) = F'(x) \Rightarrow$ pro $x \leqq 2$: $f(x) = 0$

pro $x \geqq 10$: $f(x) = 0$

pro $x \in (2,10)$: $f(x) = \frac{-1}{32}(x-10) = \frac{1}{32}(10-x)$

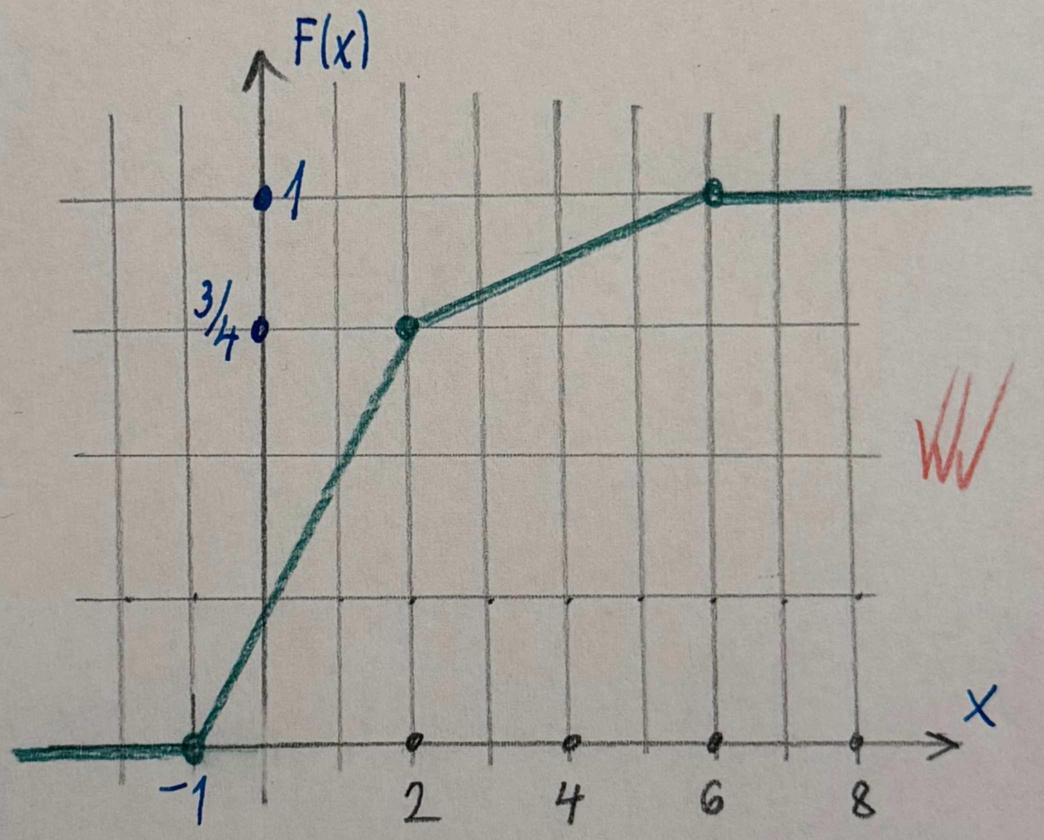
$f(2) = \frac{8}{32} = \frac{1}{4}$

$f(10) = 0$



$S = \frac{1}{2} \cdot \frac{1}{4} \cdot 8 = 1$

44)



Hustota prsti:

$$f(x) = \begin{cases} 0 & x \leqq -1 \text{ nebo } x \geqq 6 \\ \frac{1}{4} & x \in (-1,2) \\ \frac{1}{16} & x \in \langle 2,6) \end{cases}$$

nebo doplnění funkční hodnoty v levém obrázku

A5)

Normalizace: $\sum_{n \in E} P[X=n] = \sum_{n=0}^{\infty} A q^{2n} = A \frac{1}{1-q^2} \overset{!}{=} 1$

$$\Rightarrow A = 1-q^2$$

Střední hodnota:

$$E(X) = \sum_{n \in E} n \cdot P[X=n] = \sum_{n=0}^{\infty} An \cdot q^{2n} = \left\| \begin{array}{l} \sum_{n=0}^{\infty} q^{2n} = \frac{1}{1-q^2} \quad \Big/ \frac{d}{dq} \\ \sum_{n=0}^{\infty} 2n \cdot q^{2n-1} = \frac{2q}{(1-q^2)^2} \quad \Big/ \cdot q \cdot \frac{1}{2} \\ \sum_{n=0}^{\infty} n\, q^{2n} = \frac{q^2}{(1-q^2)^2} \end{array} \right\| =$$

$$= A \cdot \frac{q^2}{(1-q^2)^2} = \frac{q^2}{1-q^2} \overset{!}{=} \frac{1}{8}$$

$$8q^2 = 1 - q^2$$

$$9q^2 = 1$$

$$q = 1/3$$

B1)

– kopie příkladu A5) $\quad E(X) = \dfrac{q^2}{1-q^2} \stackrel{!}{=} \dfrac{4}{5} \Rightarrow 4q^2 + 5q^2 - 4 = 0$

$\Rightarrow 9q^2 = 4 \Rightarrow q = \dfrac{2}{3}$

B2)

$E = \{1, 2, 3, \dots, 36\}$

$\mathrm{card}(E) = 36$

$A = \{1, 2, 3, 34, 35, 36\} \qquad P[X \in A] = \dfrac{\mathrm{card}(A)}{\mathrm{card}(E)} = \dfrac{6}{36} = \dfrac{1}{6}$

$F(27{,}113) = P[X < 27{,}113] = P[X \in \{1, 2, 3, \dots, 27\}] =$

$= \dfrac{\mathrm{card}(\hat{27})}{\mathrm{card}(E)} = \dfrac{27}{36} = \dfrac{3}{4} = 75\,\%$

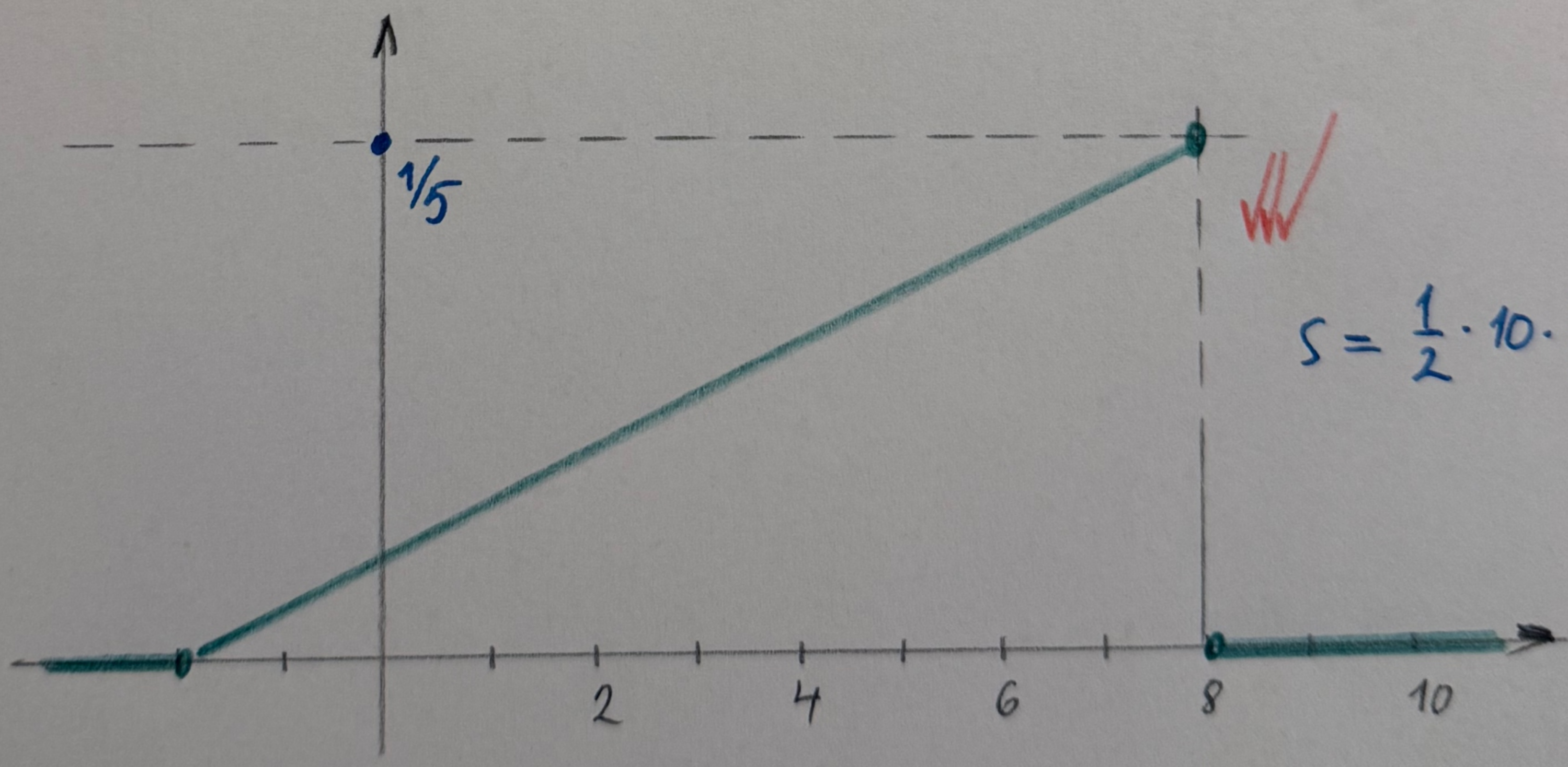
B3)

Pro $x \leq -2$: $\quad f(x) = F'(x) = 0' = 0$

Pro $x \in (-2, 8)$: $\quad f(x) = F'(x) = \left(\dfrac{1}{100}(x+2)^2\right)' = \dfrac{1}{50}(x+2)$

Pro $x \geq 8$: $\quad f(x) = 1' = 0$

navíc: $\displaystyle\lim_{x \to -2} \frac{1}{50}(x+2) = 0$ & $\displaystyle\lim_{x \to 8} \frac{1}{50}(x+2) = \frac{1}{5}$



$S = \dfrac{1}{2} \cdot 10 \cdot \dfrac{1}{5} = \underline{1}$

$C \cup D = \langle -1, 7) \quad \& \quad C \setminus D = \langle -1, 1)$

$P[X \in C \cup D] = P[X \in \langle -1, 7)] = F(7) - F(-1) = \dfrac{81}{100} - \dfrac{1}{100} = \dfrac{4}{5}$

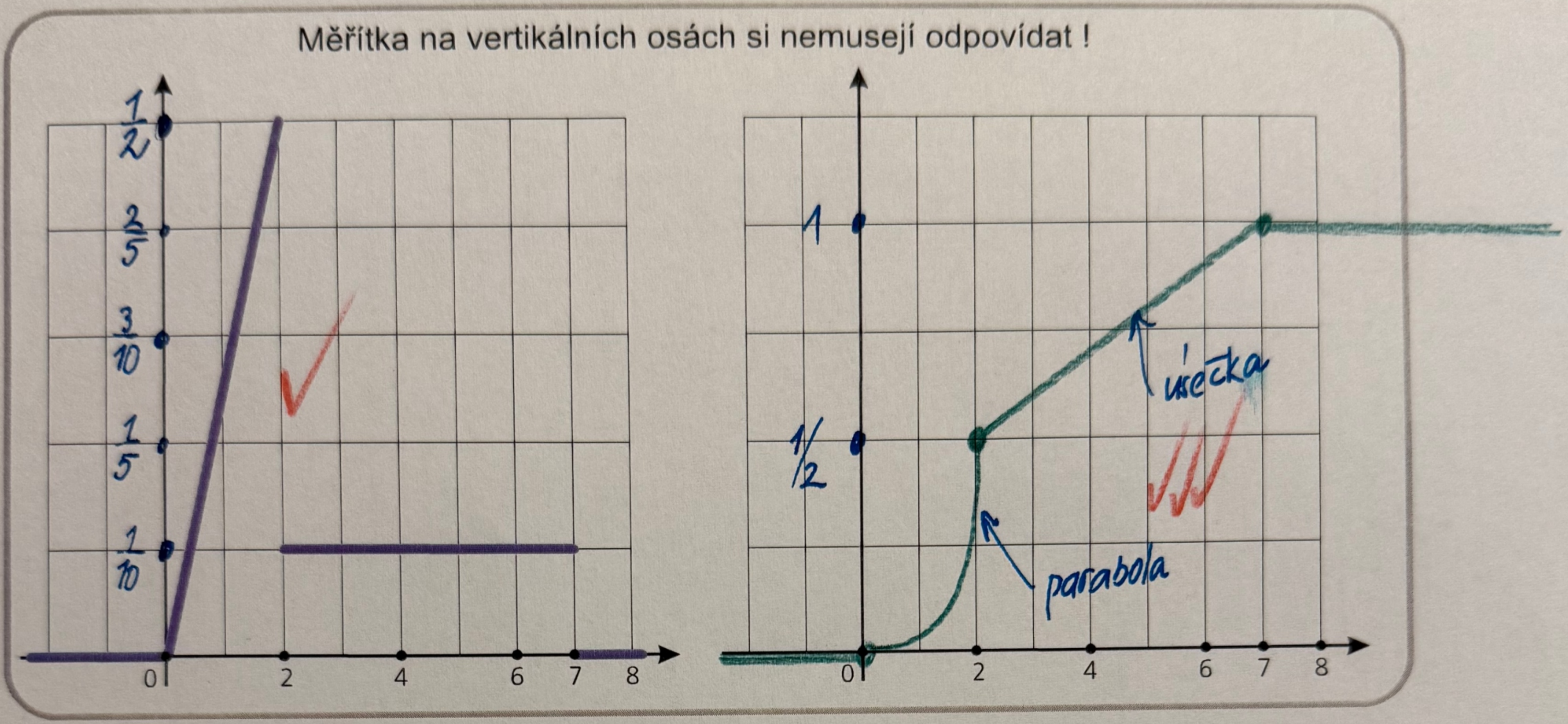
$P[X \in C \setminus D] = P[X \in \langle -1, 1)] = F(1) - F(-1) = \dfrac{9}{100} - \dfrac{1}{100} = \dfrac{2}{25}$

Hustota: $x \leqq 0 \ldots f(x)=0$
$x \in (0,2) \ldots f(x) = \frac{1}{4}x$
$x \in (2,7) \ldots f(x) = 1/10$
$x \geqq 7 \ldots f(x) = 0$

Distrib. funkce: $x \leqq 0 \ldots F(x)=0$
$x \in (0,2) \ldots F(x) = \int_0^x \frac{1}{4} y \, dy = \frac{x^2}{8}$
$x \in (2,7) \ldots F(x) = \frac{1}{2} + \int_2^x 1/10 \, dy = \frac{1}{2} + \frac{x-2}{10}$
$x \geqq 7 \ldots F(x) = 1$

4 (5 bodů)

K hustotě pravděpodobnosti z obrázku vykreslete (do volného obrázku vpravo) odpovídající distribuční funkci. Doplňte také měřítka na vertikálních osách v obou obrázcích.



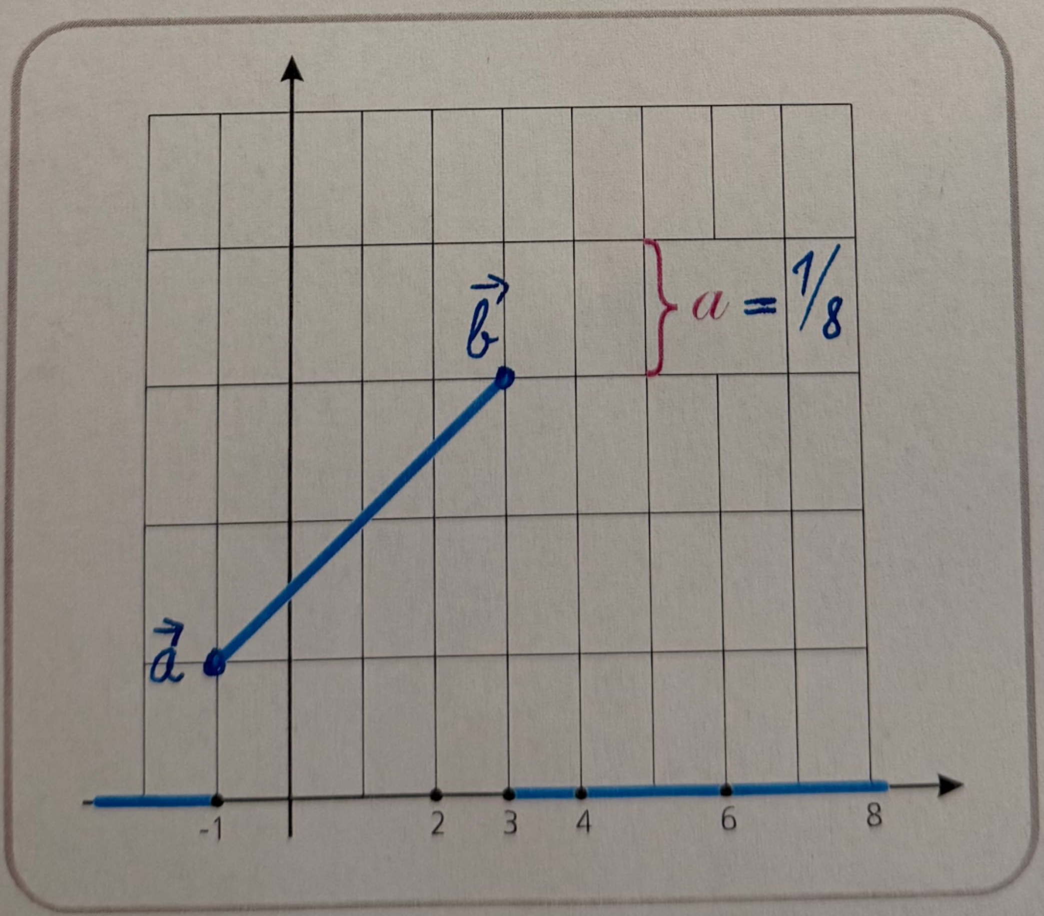
5 (7 bodů)

Na obrázku je vykreslena hustota pravděpodobnosti náhodné proměnné $\mathbb{X}$. Zodpovězte následující dotazy:

- Jakou hodnotu má číslo a?
- Jaký je analytický předpis této hustoty?
- Co je střední hodnotou zkoumané náhodné proměnné?

Obsah: $S = 4a + \frac{1}{2} \cdot 4 \cdot 2a =$
$= 8a \overset{!}{=} 1$
$a = 1/8$

$\vec{a} = (-1, \frac{1}{8})$
$\vec{b} = (3; \frac{3}{8})$
$y = k \cdot x + q$
$\frac{1}{8} = -k + q$
$\frac{3}{8} = 3k + q$
$k = \frac{1}{16}$ & $q = \frac{3}{16}$
$y = \frac{1}{16}x + \frac{3}{16}$



$$E(X) = \int_{-1}^{3} x \left(\frac{x}{16} + \frac{3}{16} \right) dx = \int_{-1}^{3} \left(\frac{x^2}{16} + \frac{3x}{16} \right) dx = \frac{1}{3 \cdot 16} \left[x^3 \right]_{-1}^{3} + \frac{3}{32} \left[x^2 \right]_{-1}^{3} =$$

$$= \frac{1}{3 \cdot 16} \cdot 28 + \frac{3}{32} \cdot 8 = \frac{1}{3 \cdot 4} 7 + \frac{3}{4} = \frac{7}{12} + \frac{9}{12} = \frac{16}{12} = \frac{4}{3}$$