$f(x) = x \qquad g_a(x) = -3x^2 + x + a$

8 bodů

$$\langle f | h \rangle = \int_{-1}^{2} f(x)\, h(x)\, dx$$

$$\| f \|^2 = \langle f | f \rangle = \int_{-1}^{2} f^2(x)\, dx$$

$$\rho^2(f, g_a) = \| f - g_a \|^2 = \int_{-1}^{2} (f(x) - g_a(x))^2\, dx =$$

$$= \int_{-1}^{2} (x + 3x^2 - x - a)^2\, dx = \int_{-1}^{2} (3x^2 - a)^2\, dx =$$

$$= \int_{-1}^{2} (9x^4 - 6x^2 a + a^2)\, dx = \left[\frac{9}{5} x^5 - 2x^3 a + a^2 x \right]_{-1}^{2} =$$

$$= \frac{9}{5}(32+1) - 2a(8+1)a + a^2(2+1) = \frac{297}{5} - 18a + 3a^2$$

Hledáme minimum funkce $H(a) = 3a^2 - 18a + \frac{297}{5}$

$$H'(a) = 6a - 18 \overset{!}{=} 0$$

$$a = 3$$

… zcela jistě jde o minimum

Nejblíže k funkci $f(x) = x$ má funkce $\underbrace{g(x) = -3(x^2-1) + x}$.

$$M_a = \{(x,y) \in \mathbb{R}^2 : x = ay^2\}$$

$$\lim_{\substack{(x,y)\to\vec{0} \\ (x,y)\in M_a}} g(x,y) = \lim_{y\to 0} \frac{6\cdot a\cdot y^4}{a^2y^4 + 9y^4} = \frac{6a}{a^2+9} \overset{!}{=} -1$$

$$a^2 + 9 = -6a$$

$$a^2 + 6a + 9 = 0$$

$$(a+3)^2 = 0$$

$$a = -3$$

Hledaná posloupnost:

$$\vec{a}_n = \begin{pmatrix} -\frac{3}{n^2} \\ \frac{1}{n} \end{pmatrix}$$

$$\langle \vec{x} | \vec{y} \rangle = \vec{x}^T \begin{pmatrix} 4 & -3 \\ -3 & 3 \end{pmatrix} \vec{y}$$

$$\|\vec{u}\|^2 = \langle \vec{u} | \vec{u} \rangle = 4u_1^2 + 3u_2^2 - 6u_1u_2 = 36 + 48 - 72 = 12$$

$$\|\vec{v}\|^2 = \langle \vec{v} | \vec{v} \rangle = 4v_1^2 + 3v_2^2 - 6v_1v_2 = 3$$

$$\|\vec{u}\| = \sqrt{12} \quad \& \quad \|\vec{v}\| = \sqrt{3}$$

$$\langle \vec{u} | \vec{v} \rangle = (3,4) \begin{pmatrix} 4 & -3 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} = (3,4) \begin{pmatrix} 3 \\ -3 \end{pmatrix} = 9 - 12 = -3$$

$$\cos(\alpha) = \frac{|\langle \vec{u} | \vec{v} \rangle|}{\|\vec{u}\| \cdot \|\vec{v}\|} = \frac{3}{\sqrt{12} . \sqrt{3}} = \frac{3}{6} = \frac{1}{2}$$

$$\underline{\alpha = 60^\circ}$$

znaménko → 4,5
(1. ℓ°, 2. $\sphericalangle > \pi/2$?)

$$\frac{\partial f}{\partial x}(0,0) = \lim_{\tau\to 0} \frac{3\tau+5-5}{\tau} = 3$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{\tau\to 0} \frac{-\frac{4\tau^3}{\tau^2}+\tau+5-5}{\tau} = -3$$

$\Rightarrow$ grad $f(0,0) = (3,-3)$

$$\eta(x,y) = -df_a(x,y) + f(x,y) - f(0,0)$$

$$\eta(x,y) = -3x + 3y + 3x - \frac{4y^3}{x^2+y^2} + y + 5 - 5$$

$$\eta(x,y) = 4y - \frac{4y^3}{x^2+y^2} = \underbrace{\frac{4x^2y}{x^2+y^2}}_{\text{tvar zbytkové funkce}}$$

$$\lim_{(x,y)\to(0,0)} \frac{\eta(x,y)}{\|(x,y)\|} = \lim_{(x,y)\to\vec{0}} \frac{4x^2y}{x^2+y^2}\cdot\frac{1}{\sqrt{x^2+y^2}} =$$

$$= \lim_{(x,y)\to\vec{0}} \frac{4x^2y}{(x^2+y^2)^{3/2}}$$ neexistuje

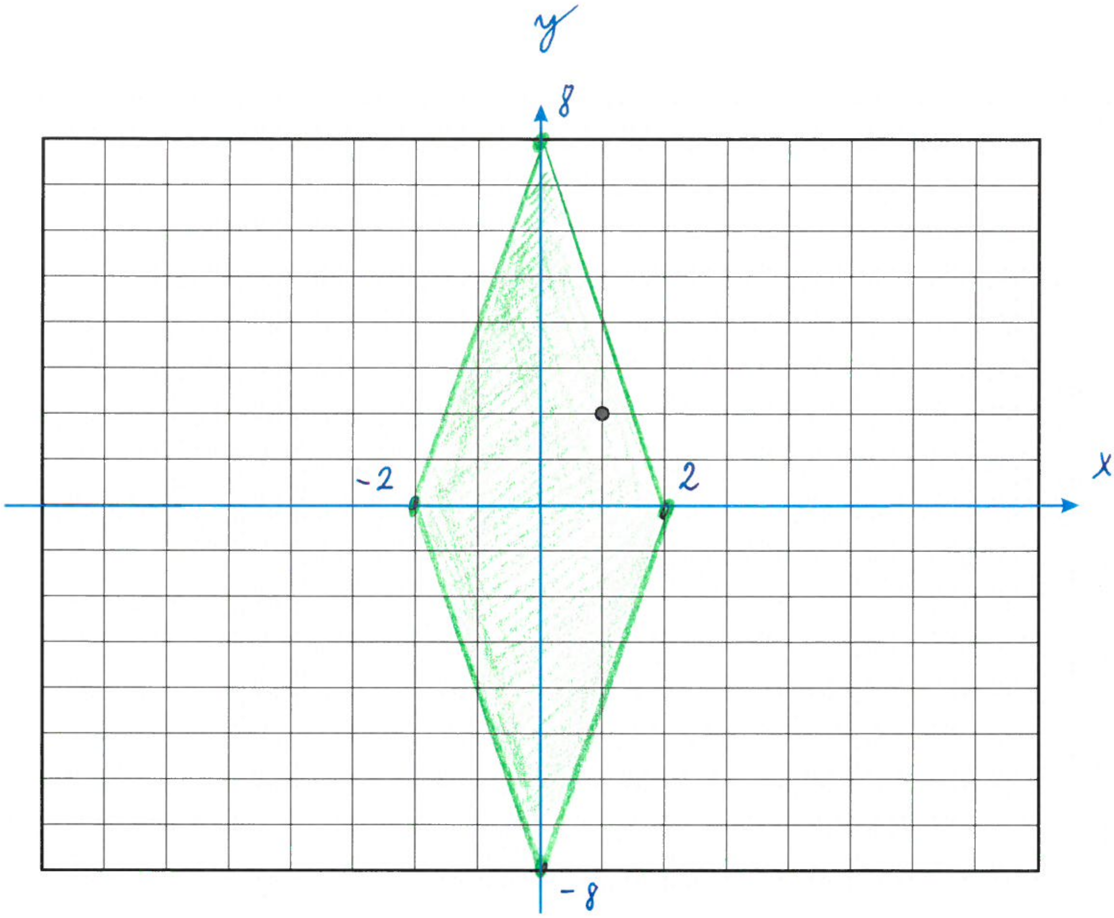
zdůvodnění

nemůže existovat

$$M_a = \{(x,y)\in\mathbb{R}^2 : y = ax\}$$

$$\lim_{\substack{(x,y)\to\vec{0}\\ y=ax}} \frac{4x^2y}{(x^2+y^2)^{3/2}} = \lim_{x\to 0} \frac{4x^2\cdot a\cdot x}{(x^2+a^2x^2)^{3/2}} = \frac{4a}{(1+a^2)^{3/2}}$$

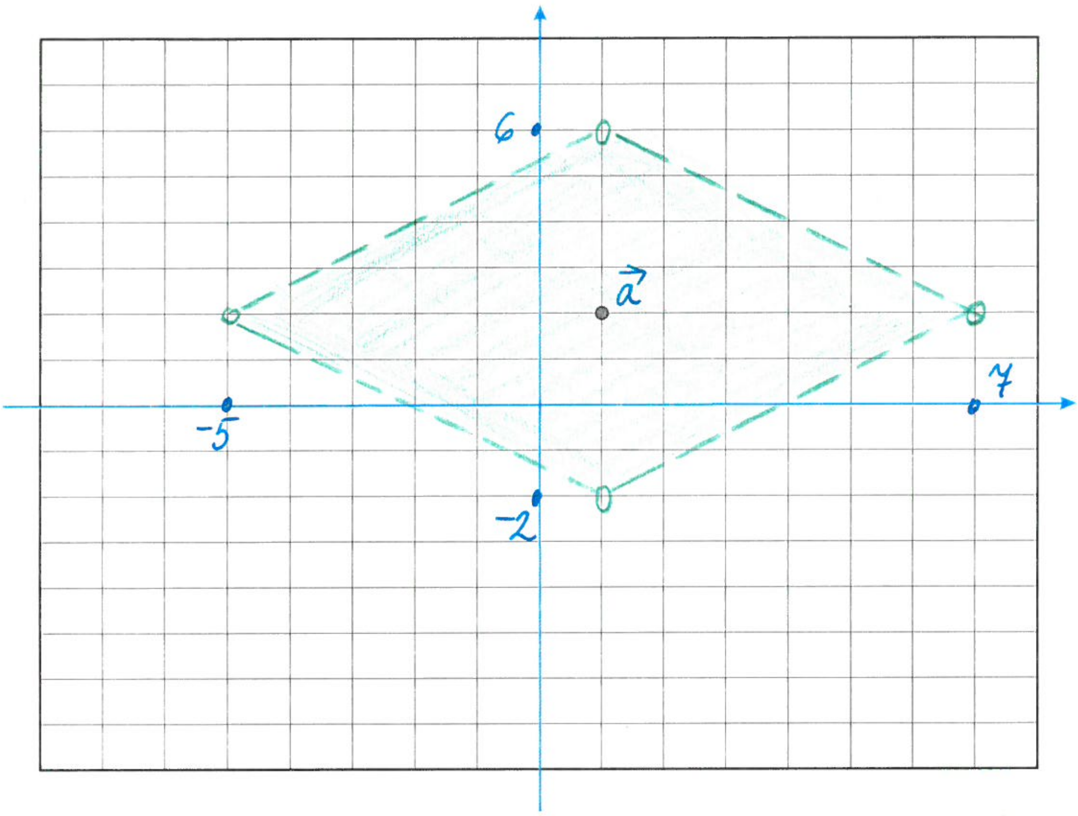
to závisí na hodnotě $\underline{\underline{a}}$



okolí:

pro $\varepsilon < 1$

$\Rightarrow$ body $(0,8)$ & $(0,-8)$, protože v jejich okolí o poloměru $\varepsilon < 1$ leží jen jediný bod množiny A



$\|\vec{x}\| = 2|x_1| + 3|x_2|$ $\qquad\qquad$ $\vec{a} = (1,2)$

$\rho(\vec{x},\vec{y}) = \|\vec{x} - \vec{y}\| = 2|x_1 - y_1| + 3|x_2 - y_2|$

$\rho(\vec{x},\vec{a}) = 2|x_1 - 1| + 3|x_2 - 2| < 12$

$x_2 = 2 \Rightarrow \quad 2|x_1 - 1| < 12 \Leftrightarrow |x_1 - 1| < 6$

$x_1 = 1 \Rightarrow \quad 3|x_2 - 2| < 12 \Leftrightarrow |x_2 - 2| < 4$

$x_1 > 1 \wedge x_2 > 2 \Rightarrow \quad 2(x_1 - 1) + 3(x_2 - 2) < 12$

$3(x_2 - 2) < 12 - 2(x_1 - 1)$

$\underbrace{x_2 < 6 - \frac{2}{3}(x_1 - 1) + 2}_{\text{hraniční křivka je část přímky}}$