9b

$x = \rho\sqrt{\cos\varphi}\cdot a$
$y = \rho\sqrt{\sin\varphi}\cdot b$

volba mojich souřadnic

$x \geq 0 \wedge y \geq 0 \Rightarrow \varphi \in (0, \tfrac{\pi}{2})$

$\left(\frac{x^4}{a^4} + \frac{y^4}{b^4}\right)^{4/3} \leq 2\left(\frac{xy}{ab}\right)^2 \Rightarrow \rho^{16/3} \leq 2\rho^4 \cos\varphi\sin\varphi$

$\rho^{4/3} \leq \underbrace{\sin 2\varphi}$

>0 na $(0, \tfrac{\pi}{2})$
$\Rightarrow$ žádné jiné omezení na φ není nutné

$$\mu_2(Q) = \int_Q 1\, d\mu(x,y) = \left| \begin{array}{l} \frac{d\varphi}{dx} = \frac{d}{dx}(x^3) = 3x^2 \\ \frac{d\psi}{dy} = \frac{d}{dy}(y^3) = 3y^2 \end{array} \right| =$$

převod na klasický integrál

$$= 9\int_Q x^2y^2\, d\lambda(x,y) = \left| \begin{array}{l} x = a\rho\sqrt{\cos\varphi} \\ y = b\rho\sqrt{\sin\varphi} \end{array} \quad d\lambda(x,y) = \overbrace{\frac{1}{2}ab\rho\frac{1}{\sqrt{\cos\varphi\sin\varphi}}}^{|J|} d(\varphi,\rho) \right| =$$

$$= 9\cdot\int_0^{\pi/2}\int_0^{(\sin 2\varphi)^{3/4}} a^2b^2\rho^4\cos\varphi\sin\varphi\cdot\frac{1}{2}ab\rho\frac{1}{\sqrt{\cos\varphi\sin\varphi}}\, d\rho\, d\varphi =$$

!!

meze musejí být OK a musí být $|J|$

$$= \frac{9}{2}\cdot\frac{1}{6}a^3b^3\int_0^{\pi/2}\sqrt{\cos\varphi\sin\varphi}\cdot\left[\rho^6\right]_0^{(\sin 2\varphi)^{3/4}} d\varphi =$$

$$= \frac{3}{4}a^3b^3\int_0^{\pi/2}\sqrt{\cos\varphi\sin\varphi}\,(\sin 2\varphi)^{9/2}\, d\varphi = \frac{3}{4}\sqrt{2}\cdot 16\cdot\int_0^{\pi/2}(\cos\varphi\sin\varphi)^5\, d\varphi =$$

$$= \frac{3}{4}a^3b^3\frac{1}{\sqrt{2}}\int_0^{\pi/2}\sin(2\varphi)\cdot\sin^4(2\varphi)\, d\varphi = \frac{3}{4}a^3b^3\frac{1}{\sqrt{2}}\int_0^{\pi/2}\sin(2\varphi)\cdot\left[1-\cos^2(2\varphi)\right]^2 d\varphi$$

$$= \left| \begin{array}{l} u = \cos(2\varphi) \\ du = -2\sin(2\varphi)\, d\varphi \end{array} \right| = \frac{3}{8}\frac{1}{\sqrt{2}}a^3b^3\int_{-1}^{1}(1-u^2)^2\, du =$$

$$= \frac{3}{4}\frac{1}{\sqrt{2}}a^3b^3\int_0^1(1-2u^2+u^4)\, du = \frac{3}{4}\frac{1}{\sqrt{2}}a^3b^3\left[u - \frac{2}{3}u^3 + \frac{u^5}{5}\right]_0^1 =$$

$$= \frac{3}{4}\frac{1}{\sqrt{2}}a^3b^3\left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{3}{4}\frac{1}{\sqrt{2}}a^3b^3\,\frac{15-10+3}{15} = \underline{\frac{\sqrt{2}}{5}a^3b^3}$$

$$H(\beta) = \int_0^\infty \frac{\ln(x^2+\beta^2) - \ln(x^2+1)}{x^2+\alpha^2}\,dx$$

(10 bodů celkem)

Hlavní předpoklad věty o derivaci integrálu podle parametru:

$$\left|\frac{\partial g(x,\beta)}{\partial \beta}\right| = \left|\frac{2\beta}{(x^2+\alpha^2)(x^2+\beta^2)}\right| \leq \frac{2\beta}{(x^2+\alpha^2)\beta^2} = \frac{2}{(x^2+\alpha^2)\beta} \leq$$

$$\overset{\beta \geq 1}{\leq} \underline{\frac{2}{x^2+\alpha^2}}$$

nesmí záviset na β!

← o této funkci ukážeme, že je integrabilní

$$\int_0^\infty \frac{2}{x^2+\alpha^2}\,dx = \frac{2}{\alpha^2}\int_0^\infty \frac{1}{1+\left(\frac{x}{\alpha}\right)^2}\,dx = \left|\begin{matrix} u = \frac{x}{\alpha} \\ dx = \alpha\,du \end{matrix}\right| = \frac{2}{\alpha}\left[\operatorname{arctg}\frac{x}{\alpha}\right]_0^\infty =$$

$$= \frac{2}{\alpha}\,\frac{\pi}{2} = \frac{\pi}{\alpha} \quad \Rightarrow \quad \underline{\frac{2}{x^2+\alpha^2} \in \mathcal{L}(0,+\infty)}$$

prokázání integrability

Výpočet:

$$\frac{\partial H}{\partial \beta} = \int_0^\infty \frac{2\beta}{(x^2+\alpha^2)(x^2+\beta^2)}\,dx = 2\beta\int_0^\infty \left(\frac{Ax+B}{x^2+\alpha^2} + \frac{Cx+D}{x^2+\beta^2}\right)dx =$$

← správný tvar parciálních zlomků

$$= \left|\begin{matrix} (A,C) = (0,0) \\ B = \frac{1}{\beta^2-\alpha^2} \quad \& \quad D = \frac{-1}{\beta^2-\alpha^2} \end{matrix}\right| = \frac{2\beta}{\beta^2-\alpha^2}\int_0^\infty \frac{1}{x^2+\alpha^2}\,dx -$$

← hodnoty konstant (obou)

$$- \frac{2\beta}{\beta^2-\alpha^2}\int_0^\infty \frac{1}{x^2+\beta^2}\,dx = \frac{2\beta}{\beta^2-\alpha^2}\left(\frac{\pi}{2\alpha} - \frac{\pi}{2\beta}\right) = \frac{\beta}{\beta^2-\alpha^2}\cdot\frac{\pi(\beta-\alpha)}{\alpha\beta} =$$

$$= \underline{\frac{\pi}{\alpha(\alpha+\beta)}}$$

$$H(\beta) = \int \frac{\pi}{\alpha(\alpha+\beta)}\,d\beta = \underline{\frac{\pi}{\alpha}\ln(\alpha+\beta) + C}$$

$$H(\beta=1) = \underline{\frac{\pi}{\alpha}\ln(\alpha+1) + C \overset{!}{=} 0} \quad \Rightarrow \quad C = -\frac{\pi}{\alpha}\ln(\alpha+1)$$

$$\underline{H(\beta) = \frac{\pi}{\alpha}\ln\frac{\alpha+\beta}{\alpha+1} = \frac{\pi}{\alpha}\ln\left(1+\frac{\beta-1}{\alpha+1}\right)}$$

4b

2	-1	2	-1
-1	2	-1	2
2	-1	2	-1
-1	2	-1	2

funkční hodnoty

$\psi(y) = y$

y				
4–3	1	3	5	7
3–2	1	3	5	7
2–1	1	3	5	7
1–0	1	3	5	7
x: 0 1 2 3 4				

hodnoty měr jednotlivých políček

$\rightarrow \varphi(x) = x \cdot |x|$

Příklad:

$$\mu_2(\langle 2,3) \times \langle 2,3)) = (9-4)\cdot(3-2) =$$
$$= 5 \cdot 1 = 5$$

+2	-3	10	-7
-1	6	-5	14
2	-3	10	-7
-1	6	-5	14

součin $g(x,y) \cdot \mu_2(\square)$

✓ za smysluplné úvahy

$$\int_{\mathbb{R}^2} g(x,y)\, d\mu(x,y) = \int_{supp(g)} g(x,y)\, d\mu(x,y) =$$

$$= \int_{\langle 0,4)^2} g(x,y)\, d\mu(x,y) = 2 + 6 + 10 + 14 = \underline{\underline{22}}$$

✓

$x = a\rho\cos^4\varphi$
$y = b\rho\sin^4\varphi$
pseudopolární souřadnice

10b

Rovnice křivky v kart. souřadnicích: $\left(\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}\right)^8 = 16\frac{xy}{ab}$

Rovnice křivky v pseudopolárních souřadnicích:

$$\rho = (\sin(2\varphi))^2$$

$x, y > 0$ (zadání) $\Rightarrow \varphi \in \langle 0; \pi/2 \rangle$

Křivka je jednoduchá, uzavřená a hladká regulární (viz příklad 5), a proto bude možno užít Greenovy věty: citace předpokladů

$$\int_C \vec{F}(x,y)\, d\mu_e(x,y) = \int_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) d(x,y)$$

(při kladném proběhnutí křivky)

$$\int_C (x^2-y;\, x+y^3)\, d\mu_e(x,y) = \left| \begin{array}{ll} F_1(x,y) = x^2-y & \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 1+1 = 2 \\ F_2(x,y) = x+y^3 & \end{array} \right| =$$

aplikace G-věty

$$= 2\int_S 1 \cdot d(x,y) = \left| \begin{array}{ll} x = a\rho\cos^4\varphi & d(x,y) = 4ab\rho\cos^3(\varphi)\sin^3(\varphi)\, d(\rho,\varphi) \\ y = b\rho\sin^4\varphi & \end{array} \right| =$$

$$= 2\cdot 4ab \int_0^{\pi/2}\int_0^{\sin^2(2\varphi)} \rho\cos^3(\varphi)\sin^3(\varphi)\, d\rho\, d\varphi = 4ab\int_0^{\pi/2} \sin^4(2\varphi)\cdot\cos^3(\varphi)\sin^3(\varphi)\, d\varphi =$$

$$= \frac{1}{2}ab\int_0^{\pi/2} \sin^7(2\varphi)\, d\varphi = \left| \begin{array}{l} u = 2\varphi \\ du = 2d\varphi \end{array} \right| = \frac{1}{4}ab\int_0^{\pi} \sin^7(u)\, du =$$

$$= \frac{1}{4}ab\int_0^{\pi} (1-\cos^2 u)^3 \cdot \sin(u)\, du = \left| \begin{array}{l} z = \cos u \\ dz = -\sin(u)\, du \end{array} \right| =$$

$$= \frac{1}{4}ab\int_{-1}^{1} (1-z^2)^3\, dz = \frac{1}{2}ab\int_0^1 (1-3z^2+3z^4-z^6)\, dz =$$

$$= \frac{1}{2}ab\left(1-1+\frac{3}{5}-\frac{1}{7}\right) = \frac{1}{2}ab\,\frac{21-5}{35} = \frac{8}{35}ab$$

4b

$x = a\rho\cos^4(\varphi)$
$y = b\rho\sin^4(\varphi)$ } pomocné plošné souřadnice

$$\left(\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}\right)^8 = \frac{16xy}{ab}$$ ← rovnice křivky v kartézských souřadnicích

$$\rho^4 = 16\rho^2\cos^4(\varphi)\sin^4(\varphi)$$

$$\rho = \sin^2(2\varphi)$$ ← rovnice křivky v pseudopolárních souřadnicích

Křivková parametrizace:

$$\vec{\varphi}(t) = \left(\sin^2(2t)\cdot\cos^4(t)\cdot a\ ;\ b\cdot\sin^2(2t)\cdot\sin^4(t)\right)$$

✓ parametrizace jako vektor o jednom parametru

$$t \in \langle 0; \pi/2 \rangle \Leftarrow x, y \overset{!}{>} 0 \text{ (viz zadání)}$$

✓ vynesení intervalu

počáteční bod: $\vec{\varphi}(0) = (0,0)$

koncový bod: $\vec{\varphi}(\pi/2) = (0,0)$

$\vec{\varphi}(0) = \vec{\varphi}(\pi/2) \Rightarrow$ je naplněna podmínka z definice uzavřenosti křivky C
$\Rightarrow$ C je uzavřená ✓

$$\dot{\vec{\varphi}}(t) = \left(4\sin(2t)\cos(t\cdot 2)\cdot\cos^4(t)\,a - 4_a\sin^2(2t)\cos^3(t)\sin t\ ;\right.$$
$$\left.4b\sin(2t)\cos(2t)\cdot\sin^4(t) + 4b\sin^2(2t)\sin^3(t)\cos t\right) \overset{!}{\neq} (0,0)$$

na $(0;\pi/2)$ zde jen otevřený interval!!!

První složka:

$$\cancel{4\sin(2t)}\cos(2t)\cancel{\cos^3(t)}\cos(t) - \cancel{4\sin(2t)}\sin(2t)\cancel{\cos^3(t)}\sin t \overset{?}{=} 0$$

$$\cos(2t)\cos(t) - \sin(2t)\sin(t) = 0$$ ← součtový vzorec

$$\cos(3t) \overset{!}{=} 0$$

$$3t = \frac{\pi}{2} \Rightarrow t = \pi/6$$ ✓

Druhá složka pro $t = \pi/6$:

všechny členy kladné

$$\sin(\pi/3)\cdot\cos(\pi/3)\sin^4(\pi/6) + \sin^2(\pi/3)\sin^3(\pi/6)\cos(\pi/6) > 0$$

$$\Rightarrow \forall t \in (0;\pi/2): \|\dot{\vec{\varphi}}\| \neq 0 \Rightarrow$$ C je hladká regulární

10b

$$\int_S (z^3;\, 0;\, xy)\, d\mu_S(x,y,z)$$

$$S = \left\{ (x,y,z) \in \mathbb{R}^3 :\ \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 + \frac{z^4}{c^4} = 1 \ \wedge\ x,y,z > 0 \right\}$$

$$\left.\begin{aligned} x &= a\sqrt{\cos\vartheta}\,\cos\varphi \\ y &= b\sqrt{\cos\vartheta}\,\sin\varphi \\ z &= c\sqrt{\sin\vartheta} \end{aligned}\right\} \checkmark$$

! Gaussova věta užít nelze ⇐ S není jednoduše uzavřená!

$$\frac{\partial \vec{f}}{\partial \varphi} = \left(-a\sqrt{\cos\vartheta}\,\sin\varphi;\ b\sqrt{\cos\vartheta}\,\cos\varphi;\ 0\right) \checkmark$$

$$\frac{\partial \vec{f}}{\partial \vartheta} = \left(-\frac{a}{2}\frac{\sin\vartheta}{\sqrt{\cos\vartheta}}\cos\varphi;\ -\frac{b}{2}\frac{\sin\vartheta}{\sqrt{\cos\vartheta}}\sin\varphi;\ \frac{c}{2}\frac{\cos\vartheta}{\sqrt{\sin\vartheta}}\right) \checkmark$$

$$\begin{vmatrix} i & j & k \\ -a\sqrt{\cos\vartheta}\,\sin\varphi & b\sqrt{\cos\vartheta}\,\cos\varphi & 0 \\ -\frac{a}{2}\frac{\sin\vartheta}{\sqrt{\cos\vartheta}}\cos\varphi & -\frac{b}{2}\frac{\sin\vartheta}{\sqrt{\cos\vartheta}}\sin\varphi & \frac{c}{2}\frac{\cos\vartheta}{\sqrt{\sin\vartheta}} \end{vmatrix} =$$

$$= \left(\frac{bc}{2}\frac{\cos^{3/2}\vartheta}{\sqrt{\sin\vartheta}}\cos\varphi;\ \frac{ac}{2}\frac{\cos^{3/2}\vartheta}{\sqrt{\sin\vartheta}}\sin\varphi;\ \frac{ab}{2}\sin\vartheta\sin^2\varphi + \frac{ab}{2}\sin\vartheta\cos^2\varphi\right) =$$

$$= \left(\frac{bc}{2}\frac{\cos^{3/2}\vartheta}{\sqrt{\sin\vartheta}}\cos\varphi;\ \frac{ac}{2}\frac{\cos^{3/2}\vartheta}{\sqrt{\sin\vartheta}}\sin\varphi;\ \frac{ab}{2}\sin\vartheta\right) \checkmark\checkmark$$

$$I = \int_0^{\pi/2}\!\!\int_0^{\pi/2} \frac{bc}{2}\frac{\cos^{3/2}\vartheta}{\sqrt{\sin\vartheta}}\cos\varphi\; c^3\sin^{3/2}\vartheta\, d\vartheta\, d\varphi \checkmark + \int_0^{\pi/2}\!\!\int_0^{\pi/2} \frac{ab}{2}\sin\vartheta\; ab\cos\vartheta\sin\varphi\cos\varphi\, d\vartheta\, d\varphi \checkmark =$$

$$= \frac{bc^4}{2}\int_0^{\pi/2}\cos^{3/2}\vartheta\,\sin\vartheta\, d\vartheta \cdot \int_0^{\pi/2}\cos\varphi\, d\varphi + \frac{a^2b^2}{2}\left(\int_0^{\pi/2}\frac{1}{2}\sin 2\vartheta\, d\vartheta\right)^2 = \checkmark$$

$$= \frac{bc^4}{2}\int_0^1 u^{3/2}\, du + \frac{a^2b^2}{2}\left(\left[-\frac{1}{4}\cos 2\vartheta\right]_0^{\pi/2}\right)^2 = \frac{bc^4}{2}\cdot\frac{2}{5} + \frac{a^2b^2}{2}\,\frac{1}{4} =$$

$$= \frac{1}{5}bc^4 + \frac{1}{8}a^2b^2 \checkmark\checkmark$$

$$F(b) = \int_0^\infty e^{-ax} \frac{\sin^2(bx)}{x}\, dx$$

a) $F(b=0) = 0$

b) $x \mapsto e^{-ax} \frac{\sin^2(bx)}{x}$ měřitelná

c) $\left| \frac{\partial f}{\partial b} \right| = \left| e^{-ax} 2 \sin(bx) \cos(bx) \right| = \left| e^{-ax} \sin(2bx) \right| \leq e^{-ax} \in \mathcal{L}(0, +\infty)$

$\int_0^\infty e^{-ax} dx = \frac{1}{a} \in \mathbb{R}$ ⇑

$$\frac{\partial F}{\partial b} = \int_0^\infty e^{-ax} 2 \sin(bx) \cos(bx)\, dx = \int_0^\infty e^{-ax} \sin(2bx)\, dx = \left| 2\times \text{ p.p.} \right| =$$

nebo lépe přes komplexní integrál $\int_0^\infty e^{-ax} e^{ibx} dx$

$$= \frac{2b}{a^2 + 4b^2}$$

$$F(b) = \frac{1}{4} \ln(a^2 + 4b^2) + C$$

$$F(0) = 0 \quad \Rightarrow \quad C = -\frac{1}{4} \ln a^2$$

$$\int_0^\infty e^{-ax} \frac{\sin^2(bx)}{x}\, dx = \frac{1}{4} \ln\left(1 + 4 \frac{b^2}{a^2}\right)$$

$$\left.\begin{aligned} x &= a\rho\cos^4\varphi \\ y &= b\rho\sin^4\varphi \end{aligned}\right\}$$ pomocné plošné souřadnice

5b

$$\left(\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}\right)^8 = \frac{16xy}{ab}$$

$$\rho = \sin^2(2\varphi)$$

$$\vec{\varphi}(t) = \left(a\sin^2(2\varphi)\cos^4(\varphi);\ b\sin^2(2\varphi)\cdot\sin^4(\varphi)\right) \qquad \overset{t}{\varphi} \in \langle 0; \pi/2 \rangle$$

$$\vec{\varphi}(t) = \left(a\sin^2(2t)\cos^4(t);\ b\sin^2(2t)\cdot\sin^4(t)\right)$$

$$\dot{\vec{\varphi}}(t) = \left(4a\sin(2t)\cos(2t)\cos^4(t) - 4a\sin^2(2t)\cdot\cos^3 t\sin t;\ \text{něco}\right)$$

tečný vektor ke křivce

$$\underbrace{\langle \dot{\vec{\varphi}}(t) \mid (1,0)\rangle \overset{!}{=} 0}_{\text{matemat. přepis zadání}} \Rightarrow \dot{\varphi}_1(t) \overset{!}{=} 0$$

$$\cancel{4a}\sin\cancel{(2t)}\cos(2t)\cos^{\cancel{4}}(t) = \cancel{4a}\sin^{\cancel{2}}(2t)\cos^{\cancel{3}}(t)\sin t$$

$$\cos(2t)\cos(t) - \sin(2t)\sin(t) = 0$$

$$\cos(2t+t) = 0$$

$$\cos(3t) = 0$$

$$3t = \pi/2$$

$$\underline{\underline{t = \pi/6}}$$

$$t = \pi/6 \Rightarrow \text{bod křivky } \vec{a} = \left(a\sin^2\left(\tfrac{\pi}{3}\right)\cos^4\left(\tfrac{\pi}{6}\right);\ b\sin^2\left(\tfrac{\pi}{3}\right)\cdot\sin^4\left(\tfrac{\pi}{6}\right)\right) =$$

$$= \left(a\cdot\frac{3}{4}\cdot\frac{9}{16};\ b\cdot\frac{3}{4}\cdot\frac{1}{16}\right) = \left(\frac{27}{64}a;\ \frac{3}{64}b\right) =$$

$$\underline{\underline{= \frac{3}{64}(9a; b)}}$$

❹ (6 bodů)

Rozhodněte, zda je množinová soustava $\mathscr{A} = \{\emptyset; \{\square\}; \{\ominus\}; \{\heartsuit, \triangle\}; \{\square, \heartsuit, \triangle\}; \{\ominus, \square, \heartsuit, \triangle\}\}$ polookruhem a jak musí být navrženy chybějící hodnoty $G(\{\heartsuit, \triangle\})$ a $G(\{\ominus, \square, \heartsuit, \triangle\})$, aby množinová funkce

$$G(X) = \begin{cases} X = \emptyset & X_1 \quad \dots \quad G(X) = 0 \\ X = \{\square\} & X_2 \quad \dots \quad G(X) = 4 \\ X = \{\ominus\} & X_3 \quad \dots \quad G(X) = 7 \\ X = \{\heartsuit, \triangle\} & X_4 \quad \dots \quad G(X) = ? \\ X = \{\square, \heartsuit, \triangle\} & X_5 \dots \quad G(X) = 4 \\ X = \{\ominus, \square, \heartsuit, \triangle\} & X_6 \; . \quad G(X) = ? \end{cases}$$

6b

byla mírou na $\mathscr{A}$. Zdůvodněte!

i)

Polookruh: $\forall i, k \in \hat{6} : X_i \cap X_j \in \mathscr{A}$ (to sedí)

$(\forall i, k \in \hat{6})(\exists (X_m)_{m=1}^{M} : X_i \setminus X_j = \biguplus_{m=1}^{M} X_m)$

zde např.

$X_6 - X_4 = \{\ominus, \square\} = X_3 \cup X_2 = X_3 \uplus X_2$

$\Rightarrow$ <u>jde o polookruh</u>

✓ příklad

ii)

čemu se rovná $G(X_4)$?

$X_4 \uplus X_2 = X_5 \wedge G(X_2) = 4 \wedge G(X_5 = 4)$

z aditivity: $G(X_4 \uplus X_2) = G(X_4) + G(X_2)$

$4 = G(X_4) + 4$

$\underline{G(X_4) = 0}$

iii) $G(X_6) = ?$

$X_6 = X_5 \uplus X_3$

$G(X_6) = G(X_5) + G(X_3) = 4 + 7 = 11$

$\underline{G(X_6) = 11}$

9b

$$x = a\rho\sqrt{\cos\varphi}$$
$$y = b\rho\sqrt{\sin\varphi}$$

α) $x, y > 0 \Rightarrow \varphi \in (0; \pi/2)$

β) $$\left(\frac{x^4}{a^4} + \frac{y^4}{b^4}\right)^5 \leqq \frac{x^{12}}{a^{12}}$$

$$\rho^{20} \leqq \rho^{12}\cos^6(\varphi)$$
$$\rho^{8} \leqq \cos^6(\varphi)$$
$$\rho \leqq (\cos\varphi)^{3/4}$$

$$\mu_2(A) = \int_A 1\, d\mu(x,y) = \int_A \frac{d\varphi}{dx}\cdot\frac{d\psi}{dy}\, d(x,y) = \left\| \frac{d}{d\tau}(\tau\cdot|\tau|) = \left\langle \begin{matrix} 2\tau \dots \tau \geq 0 \\ -2\tau \quad \tau < 0 \end{matrix} \right\rangle = 2|\tau| \right\| =$$

$$= 4\int_A |xy|\, d(x,y) = 4\int_A xy\, d(x,y) = \left\| \begin{matrix} x = a\rho\sqrt{\cos\varphi} \\ y = b\rho\sqrt{\sin\varphi} \end{matrix} \quad d(x,y) = |J|\, d(\rho,\varphi) \right\| =$$

$$= 4\int_0^{\pi/2}\int_0^{(\cos\varphi)^{3/4}} a\rho\cancel{\sqrt{\cos\varphi}}\, b\rho\cancel{\sqrt{\sin\varphi}} \cdot \underbrace{\frac{1}{2}ab\rho\frac{1}{\cancel{\sqrt{\cos\varphi\sin\varphi}}}}_{|J|}\, d\rho\, d\varphi =$$

meze musí být správně! (musí být Jacobián)

$$= \frac{1}{2}4a^2b^2\int_0^{\pi/2}\int_0^{(\cos\varphi)^{3/4}} \rho^3\, d\rho\, d\varphi = \frac{1}{2}a^2b^2\int_0^{\pi/2} [\rho^4]_0^{(\cos\varphi)^{3/4}}\, d\varphi =$$

$$= \frac{1}{2}a^2b^2\int_0^{\pi/2} \cos^3(\varphi)\, d\varphi = \left\| \begin{matrix} u = \sin\varphi \\ du = \cos\varphi\, d\varphi \end{matrix} \right\| =$$

$$= \frac{1}{2}a^2b^2\int_0^1 (1-u^2)\, du = \frac{1}{2}a^2b^2\left[u - \frac{u^3}{3}\right]_0^1 = \frac{1}{2}a^2b^2\left(1 - \frac{1}{3}\right) =$$

$$= \frac{1}{2}\frac{2}{3}a^2b^2 = \frac{a^2b^2}{3}$$

$\varphi(x) = \frac{3x|x|}{x^2+4}$ $\qquad g(x) = e^{-x^2}\Theta(x-2)$ $\qquad$ 3b

$$\mathrm{supp}(g) = (2; +\infty)$$

$$\varphi(x) \in C(\mathbb{R}) \Rightarrow \mu((2;+\infty)) = \mu(\langle 2; +\infty))$$

$$\mu(\mathrm{supp}(g)) = \lim_{x\to+\infty} \varphi(x) - \varphi(2) = \lim_{x\to+\infty} \frac{3x^2}{x^2+4} - \frac{12}{8} =$$

$$= 3 - \frac{12}{8} = 3 - \frac{3}{2} = \frac{6-3}{2} = \frac{3}{2}$$